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Constant-Current Stagnation-Point Film-Anemometer Probe

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Hypersonic turbulence measurements of boundary layer stability and pressure, temperature, and velocity fluctuation levels in continuous-flow wind tunnels, such as those at AEDC, can require survey probes capable of withstanding uncommonly hostile test conditions, especially of temperature and dynamic pressure. Stagnation-point hot-film anemometer probes have demonstrated good durability in AEDC tunnels at Mach numbers 4, 6, and 8. The present report addresses questions associated with the use of such probes for the quantitative measurement of flow fluctuations in wind tunnel test applications. The steady-state and time-dependent response characteristics of the hot-film probe are analyzed. The film is assumed infinitesimally thin, subjected to a constant electric current during operation, and deposited on a substrate in which the heat flow is one-dimensional. The conduction equation in the substrate is solved simultaneously with the film power equation, which attributes changes in the film temperature to convective transfer to the flow, conductive transfer to the substrate, and ohmic heating. It is found that the film response is describable by two parameters: (1) the intrinsic (no-substrate) film time constant, and (2) a substrate loss factor which is a mix of the film, flow, and substrate properties. The flow-fluctuation sensitivities of the heated film sensor are found to be similar to those of hot wires.

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PREFACE

The work reported herein was performed by Dr. Anthony Demetriades, Bozeman, MT, for the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), in the capacity of consultant to Calspan Corporation, operating contractor for the Aerospace Flight Dynamics testing effort at the AEDC, AFSC, Arnold Air Force Base, Tennessee. The work was done under contract F40600-85-C-0020, Calspan Consultant Agreement 88-02, Work Release 88-02-02.

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ILLUSTRATIONS

1. Statement of the Problem

A "hot-film anemometer" probe is typically sketched on Figure 1. It consists of a "probe body" made of an insulator, with a very thin metallic film deposited on its blunted tip. Two conducting wires ("leadwires"), attached to the two ends of the film, enable the latter to be heated electrically by a constant external d.c. current source. If the probe is immersed in a moving fluid, then the cooling effect of the flow lowers the film temperature and hence its resistance and voltage drop. A detector connected in the circuit can then be used to associate steady-state film voltages to the steady state flow properties, or the a.c. voltage across the probe to fluctuations in these properties.

The essential questions are:

(a) How is the steady-state film voltage (or resistance) quantitatively connected, at any moment, to the steady-state flow properties?

(b) How is the film a.c. voltage magnitude quantitatively connected to the magnitude to the flow fluctuations?

(c) How is the film frequency response connected to the frequency of the flow fluctuations?

These questions will be answered in this paper for the specific case of the film lying on the stagnation region (point) of the probe, and only if the electric current is constant. For analyses of the "constant temperature" operation (where the film temperature in a fluctuating flow is forced to stay constant by appropriate variations of the current) and/or different probe geometries, one can consult the literature. For example Ling (Reference 1) and Ling and Hubbard (Reference 2) analyzed the general case of the present problem, and also the case of the film lying on a 30-deg. wedge; Lowel and Patton (Reference 3) analyzed the case of the film painted over a circular cylinder; Bellhouse and Rassmussen (Reference 4) and Blaivas and Zolkina (Reference 5) the case of the constant temperature film and so on. These various solutions are

generally different from each other because of different assumptions regarding the thermal boundary conditions and/or constraints on the film current or temperature (i.e. constant-current or constant temperature operation). The previous work closest to the present analysis is that of Ling and Hubbard in so far as the frequency response is concerned; however some differences exist which are fundamental in the final results.

All analyses (see Reference 6 for some early bibliography) including the present one, follow the same general path: first, the power-balance equation for the film is considered jointly with the conduction equation for the substrate, which is frequently assumed one-dimensional. The film equation is next reduced to an ordinary, first-order non-linear differential equation, which then is linearized by assuming that the flow fluctuations are small. This linearized equation immediately provides (a) a steady-state algebraic equation which has a simple direct solution, (b) an equally simple ordinary linear first-order equation for the film temperature fluctuations.

Another point common to all film analyses is their resemblance to the "hot wire anemometer" analysis. The latter contains a different type of conduction-loss term (Reference 7) when the end losses are considered, and no conduction loss term at all for the "ideal" hot wire; and this idealized case will be frequently mentioned under the "hot wire" terminology for comparison purposes in this report.

2. Basic Film Equations

Neglecting radiation, and referring to Figures 1 and 2 and the List of Symbols, the film power-balance equation is:

$$\frac{\epsilon}{A_f}\frac{\partial T}{\partial t} = -k_o \frac{N}{\hbar}(T-T_e) + \frac{i^2 R}{A_f} + k_s \left(\frac{\partial T_s}{\partial x}\right)_{x=0} \tag{1}$$

These terms, which have the dimensions of power flux (energy per unit area and

time) are, from left to right, the unsteady film temperature change, the convective exchange with the flow, the ohmic heating and the conductive exchange between film and substrate.

The convective term contains the Nusselt number defined so that k_o is the fluid thermal conductivity based on the flow stagnation temperature T_o, related to the adiabatic recovery temperature T_e via the recovery factor η:

$$T_e = \eta T_o \tag{2}$$

The characteristic film dimension h is the film height (Figures 1,2) which must then also be used to form the film Reynolds number

$$Re_h = \frac{\rho u h}{\mu_o} \tag{3}$$

where μ_o is the stagnation viscosity. For steady-state (ideal) hot wires only the first two terms on the r.h.s. of (1) are present; then, since in that case $A_f = A_{hw} = \pi dl$ and h becomes the wire diameter d, the hot-wire counterpart of (1) will be

$$i^2R = \pi l k_o N\ (T-T_e) \tag{4}$$

In the case of films $A_f = hw$, and if a linear resistance-temperature relation is assumed:

$$R = R_r\ (1 + \alpha T) \tag{5}$$

$$R_e = R_r\ (1 + \alpha T_e) \tag{6}$$

then eq. (1) becomes

$$\epsilon \frac{\partial T}{\partial t} = -k_o wN\ (T-T_e) + i^2 R_r\ (1+\alpha T) + k_s A_f\ \left(\frac{\partial T_s}{\partial x}\right)_{x=0} \tag{7}$$

A second equation entering the problem deals with the flow of heat in the substrate:

$$\frac{\partial T_s}{\partial t} = \beta \frac{\partial^2 T_s}{\partial x^2} \tag{8}$$

where the film is assumed infinitely thin so that $T = T_s$ $(x=0)$.

If we non-dimensionalize by

$$\bar{T} \equiv \alpha T,\ \bar{T}_e \equiv \alpha T_e,\ \bar{T}_s \equiv \alpha T_s,\ \bar{t} \equiv \frac{t\beta}{A_f},\ \bar{x} \equiv x/\sqrt{A_f} \tag{9}$$

then the film equation (7) is

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{i^2 R_r \alpha A_f}{\epsilon \beta}\ (1+\bar{T}) - \frac{wA_f k_o N}{\epsilon \beta}\ (\bar{T}-\bar{T}_e) + \frac{k_s A_f^{3/2}}{\epsilon \beta}\ \left(\frac{\partial \bar{T}_s}{\partial \bar{x}}\right)_{\bar{x}=0} \tag{10}$$

and the substrate equation becomes

$$\frac{\partial \bar{T}_s}{\partial \bar{t}} = \frac{\partial^2 \bar{T}_s}{\partial \bar{x}^2} \tag{11}$$

with boundary conditions to be discussed below.

3. Linearization of the Film Equation

Since the dependent variable $\bar{T}$ of the problem and also k_o, $\bar{T}_e$ and N are generally time-dependent, eq. (10) is non-linear. Solutions are easiest to obtain when all these quantities exhibit small fluctuations, denoted by a prime, about their mean (time-averaged) values, which appear unprimed, as follows:

$$\bar{T} = \vartheta + \vartheta'\ (\bar{t})\ ,\ \vartheta' << \vartheta \tag{12}$$

$$\bar{T}_e = \vartheta_e + \vartheta'_e\ (\bar{t})\ ,\ \vartheta'_e << \vartheta_e \tag{13}$$

$$\bar{T}_s = \vartheta_s(\bar{x}) + \vartheta'_s\ (\bar{x},\ \bar{t})\ ,\ \vartheta'_s << \vartheta_s \tag{14}$$

The convective term coefficient will for the moment be written as follows:

$$\frac{wA_f k_o N}{\epsilon B} + \frac{wA_f (k_o N)'}{\epsilon B} \equiv f + f'(\bar{t})\ ,\ f' << f \tag{15}$$

The quantity

$$B \equiv \frac{A_f i^2 \alpha R_r}{\epsilon B} \tag{16}$$

will be also assumed constant (see Section 10.4 for exceptions). If equations (12)-(16) are substituted into (10) and products of two or more small quantities are omitted, we obtain

$$\frac{\partial}{\partial \bar{t}}(\vartheta + \vartheta') = B\ (1+\vartheta+\vartheta') - f(\vartheta-\vartheta_e + \vartheta'-\vartheta'_e)$$

$$- f'(\vartheta-\vartheta_e) + \frac{k_s A_f^{3/2}}{\epsilon B}\left[\left(\frac{\partial \vartheta_s}{\partial \bar{x}}\right)_{\bar{x}=0} + \left(\frac{\partial \vartheta'_s}{\partial \bar{x}}\right)_{\bar{x}=0}\right] \tag{17}$$

4. Steady-State Solution

When $\frac{\partial}{\partial \bar{t}} = \vartheta' = \vartheta'_e = \vartheta'_s = f' = 0$, eq. (17) becomes

$$B\,(1+\vartheta) - f(\vartheta - \vartheta_e) + \frac{k_s A_f}{\epsilon B}^{3/2} \left(\frac{\partial \vartheta_s}{\partial \bar{x}}\right)_{\bar{x}=0} = 0 \qquad (18)$$

while eq. (11) becomes

$$\frac{\partial^2 \vartheta_s}{\partial \bar{x}^2} = 0 \qquad (19)$$

Eq. (19) can supply the last term in eq. (18) if two boundary conditions are available for its solution. One of these conditions is

$$\vartheta_s\,(\bar{x} = 0) = \vartheta \qquad (20)$$

The other condition is specific to the probe geometry. Often the substrate is assumed to have a finite thickness with its far face at T_e, but this condition is sometimes abused by misapplication to unsuitable geometries (Reference 4 applies this condition to a geometry in which the two faces of the substrate coincide). In the present instance this condition is suitable, as is clear from Figure 1. The probe body is seen to have a large length-to-diameter ratio; furthermore, the wetted film surface area is very much smaller than the wetted area of the probe body. Therefore, if the film is electrically heated, the probe body temperature will rise only in the vicinity of the film, out to some distance L from the latter. Beyond that the probe temperature will equal its adiabatic recovery temperature due to the flow and it is very reasonable to assume that the film and probe recovery temperatures are equal to each other and to T_e. The calculation of L will be discussed in the next Section; for the moment we will assume on the strength of the above arguments that the second boundary condition is

$$T_s\,(x = L) = T_e,\ \vartheta_s\,(\bar{x} = L/\sqrt{A_f}) = \vartheta_e \qquad (21)$$

The solution of (19) subject to conditions (20) and (21) is:

$$\vartheta_s = \vartheta + \frac{x}{L}(\vartheta_e - \vartheta) \tag{22}$$

which gives

$$\left(\frac{\partial\vartheta_s}{\partial\tilde{x}}\right)_{\tilde{x}=0} = \frac{\vartheta_e - \vartheta}{L/\sqrt{A_f}} \tag{23}$$

Substituting (23) into (18) we obtain the film non-dimensional temperature:

$$\vartheta = \frac{\vartheta_e + \dfrac{B}{f + k_s A^2{}_f/L\epsilon B}}{1 - \dfrac{B}{f + k_s A^2{}_f/L\epsilon B}} \tag{24}$$

5. <u>Discussion of the Steady-State Solution</u>

When $k_s = 0$, eqs. (5), (6) can be used to simplify eq. (24) into:

$$R = \frac{R_e}{1 - \dfrac{i^2 \alpha R_r}{k_o w N}} \tag{25}$$

This is the steady-state hot-wire equation which, if k_o is changed to k_e and w absorbed into N, is identical to eq. (14) of Reference 8.

With the substrate present ($k_s \neq 0$) it can be shown that (24) reduces to

$$R = \frac{R_e}{1 - \dfrac{i^2 \alpha R r}{k_o w N}} \tag{26}$$

where the "compound Nusselt number"

$$\bar{N} \cong N + \frac{k_s}{k_o}\frac{A_f}{wL} = N + \frac{k_s}{k_o}\frac{h}{L} \tag{27}$$

To estimate the second term on the right of (27), which is the steady-state "conduction loss" contribution to $\bar{N}$, we need to estimate the "penetration depth" L first discussed in the previous Section. Anders (Reference 9) estimates L from a simple one-dimensional steady state calculation modeling the probe as a circular cylinder parallel to and wetted by the flow, with its upstream face kept at the film temperature. He finds, setting $k_s = k_o$, that

$$L \approx \left[\frac{k_s r_p L_c}{2k_o N_p}\right]^{1/2} \tag{28}$$

If the probe characteristic length L_c is identified with the cylinder (probe body) radius r_p, then

$$\bar{N} = N + \frac{h}{r_p}\left[2N_p\left(\frac{k_s}{k_o}\right)\right]^{1/2} \tag{29}$$

For typical materials and conditions the convection-loss term on the right of (29) is of order 1-10, and may be larger than N itself.

For hot wires the conduction loss formula due to the wire supports (see Reference 10 for discussion and additional References) which corresponds to (29) is

$$\bar{N} \approx N + 4\ \frac{d^2_w}{l^2}\ \frac{k_w}{k_o} \tag{30}$$

with typical values of order 0.1 for the second term on the right, the only similarity with the second term of (29) being the appearance of the same type of conduction-loss parameter (a ratio of lengths and one of conductivities). This

parameter appears in the square root in eq. (29), and in the first power in (30), and also the film loss term contains k_s while that for the wire contains k_w. The principal difference, however, is one of magnitude: based on the numbers quoted the conduction loss for the hot wire is a mere correction to its measured value, while for the film the correction overwhelms the Nusselt number itself.

6. Unsteady Solution

6.1. The Time-Dependent Equation

When eq. (18) is subtracted from (17), the unsteady equation for the film temperature perturbation ϑ' is obtained:

$$\frac{\partial\vartheta'}{\partial t} + \vartheta'(f-B) - \frac{k_s A_f}{\epsilon B}^{3/2} \left(\frac{\partial\vartheta'_s}{\partial \bar{x}}\right)_{\bar{x}=0} = f\vartheta'_e - f'(\vartheta-\vartheta_e) \tag{31}$$

and the substrate equation becomes

$$\frac{\partial\vartheta'_s}{\partial t} = \frac{\partial^2\vartheta'_s}{\partial \bar{x}^2} \tag{32}$$

The r.h.s. of eq. (31) is the forcing function driving the linear system on the left. In principle f' and ϑ'_e are caused by flow fluctuations alone (cf. eqs. (13) and (15)). This will be shown in a later Section, but an important exception requires discussion at this early stage. Since f' is driven by N', N' by r' and r' by ϑ'_e and ϑ', a term containing ϑ' will appear on the right of eq. (31). The remainder on the r.h.s. will depend only on the flow fluctuations, assumed stationary and thus expressible as a Fourier series. For a moment we are concerned with one of the Fourier components, of frequency ω. The amplitude of this component must have the form $\bar{E}/\bar{\tau}$ where $\bar{\tau}$ is the time constant of the system (this procedure is standard for "first order" systems, which in this case means the no-substrate case). The quantity $\bar{E}$ will include the amplitude of the flow fluctuations, and its calculation will not be needed until Section 7.

Collecting the ideas expressed in the above paragraph, the flow forcing term (as can be seen considering eq. (75) to be presented later) is

$$f\theta'_e - f'(\theta-\theta_e) = \frac{\bar{E}}{\bar{\tau}}\sin\bar{\omega}\bar{t} - fN_r\theta' \tag{33}$$

so that eq. (31) is

$$\frac{\partial\theta'}{\partial t} + \theta'[f(1+N_r)-B] - \frac{k_s A_f^{3/2}}{\epsilon B}\left(\frac{\partial\theta'}{\partial \bar{x}}\right)_{\bar{x}=0} = \frac{\bar{E}}{\bar{\tau}}\sin\bar{\omega}\bar{t} \tag{34}$$

The boundary condition for this equation is to set

$$\theta' = A\sin(\bar{\omega}\bar{t} - \phi) \tag{35}$$

where A, ϕ are to be determined.

6.2 Unsteady Solution in the Absence of the Substrate

We first consider the absence of the substrate ($k_s = 0$), in which case eq. (34) becomes

$$\frac{d\theta'}{dt} + \theta'\,[f(1+N_r)-B] = \frac{\bar{E}}{\bar{\tau}}\sin(\bar{\omega}\bar{t}) \tag{36}$$

This is the familiar linear ordinary differential equation of the first order with constant coefficients, which describes the so-called "first order" systems. The inverse of the second term coefficient is the (non-dimensional) system time constant:

$$\bar{\tau} = \frac{\tau B}{A_f} = \frac{1}{f(1+N_r)-B} \tag{37}$$

The solution of (36), given in all standard texts, is

$$\theta' = \frac{\bar{E}}{[1+(\bar{\omega}\bar{\tau})^2]^{1/2}} \sin(\bar{\omega}\bar{t} - \phi) \tag{38}$$

where $\phi = \tan^{-1} \bar{\omega}\bar{\tau}$. Note that $\bar{\omega} = \frac{\omega A_f}{\beta}$, so that $\bar{\omega}\bar{t} = \omega t$.

6.3 Discussion of the Unsteady Solution Without Substrate

If we recall the definitions of f and B from equations (15) and (16), the dimensionless time constant given in (37) reduces to

$$\bar{\tau} = \frac{\epsilon\beta}{A_f k_o N w(\frac{R_e}{R} + N_r)} \tag{39}$$

and its dimensional counterpart to

$$\tau = \frac{\epsilon}{w k_o N\ (\frac{R_e}{R} + N_r)} \tag{40}$$

The time constant $\bar{\tau}$ given in (39) is the "inherent" film time constant, as if the film was "suspended in space" without a substrate. For this reason it is also the "hot wire" time constant, familiar to most readers in the case $N_r = 0$, for which we obtain the classical formulas (e.g. Reference 7, p. 241):

$$\tau = \frac{\epsilon R}{w k_o N R_e} = \frac{\epsilon\ (1+r)}{w k_o N}\ ,\ \tau\ (r=0) = \frac{\epsilon}{w k_o N} \tag{41}$$

and

$$\frac{\tau}{\tau\ (i=0)} = 1 + r \tag{42}$$

Note that in practical cases these formulas differ only slightly from the exact

expression (40), since $N_r << 1$ usually.

The bracketed factor in the denominator of (38) is the familiar attenuation due to the inherent time constant of the hot wire (or the no-substrate film). It is seen that at large ω the signal $\bar{E}$ is attenuated by 6 db/octave (a factor 10 in signal drop for a factor 10 increase in frequency); see Figures 3 and 4.

6.4 Solution of the Unsteady Problem with Substrate

To solve the complete equation (34) when $k_s \neq 0$, eq. (32) must first be solved for θ'_s. The boundary conditions for (32) are:

$$t, \bar{t} \leq 0 : \theta'_s = 0 \tag{43}$$

$$t, \bar{t} > 0 : \theta'_s (\bar{x}=0) = \theta' \tag{44}$$

In view of eq. (35), this last condition is rewritten as:

$$\theta'_s (\bar{x}=0) = \bar{A} \sin(\bar{\omega}\bar{t}-\phi) \tag{45}$$

The solution of eq. (32) with conditions (43) and (45) is a well-known one (e.g. Reference 11, p. 93 or Reference 12, p. 112) and is:

$$\theta'_s (\bar{x}, \bar{t}) = \bar{A}e^{-x\sqrt{(\frac{\bar{\omega}}{2})}} \sin(\bar{\omega}\bar{t}-\phi-\bar{x}\sqrt{\frac{\bar{\omega}}{2}}) \tag{46}$$

and

$$\left(\frac{\partial\theta'_s}{\partial\bar{x}}\right)_{\bar{x}=0} = -\bar{A}\sqrt{\frac{\bar{\omega}}{2}}[\sin (\bar{\omega}\bar{t} - \phi) + \cos(\bar{\omega}\bar{t} - \phi)] \tag{47}$$

With the aid of (35), (37) and (47), eq. (34) becomes

$$\bar{A}\bar{\omega}\cos(\bar{\omega}\bar{t} - \phi) + \frac{\bar{A}}{\bar{\tau}} \sin(\bar{\omega}\bar{t} - \phi)$$

$$+ \frac{k_s A_f^{3/2}}{\epsilon B} \bar{A}\sqrt{\frac{\bar{\omega}}{2}}[\sin(\bar{\omega}\bar{t} - \phi) + \cos(\bar{\omega}\bar{t} - \phi)] = \frac{\bar{E}}{\bar{\tau}} \sin\bar{\omega}\bar{t} \tag{48}$$

By zeroing the coefficients of $\sin\bar{\omega}\bar{t}$ and $\cos\bar{\omega}\bar{t}$ we solve for $\bar{A}$ and ϕ for insertion into eq. (35); the result for the non-dimensional film perturbation θ' and phase angle ϕ is:

$$\theta'(\bar{t}) = \alpha T'(\bar{t}) = \frac{\bar{E}\sin(\bar{\omega}\bar{t}-\phi)}{[(\bar{\omega}\bar{\tau}+Q\sqrt{\bar{\omega}\bar{\tau}})^2 + (1+Q\sqrt{\bar{\omega}\bar{\tau}})^2]^{1/2}} \tag{49}$$

$$\phi = \tan^{-1} \frac{\bar{\tau}\bar{\omega} + Q\sqrt{\bar{\omega}\bar{\tau}}}{1 + Q\sqrt{\bar{\omega}\bar{\tau}}} \tag{50}$$

where $\bar{\tau}$ is as already given in (39) and Q, which will be called the "conduction loss factor" or simply "loss factor" is

$$Q = \frac{k_s A_f^{3/2}}{\epsilon B} \left[\frac{\bar{\tau}}{2}\right]^{1/2} \tag{51}$$

An alternative form of Q, useful for practical computations, is given in Section 10.2.

6.5 Discussion of the Unsteady Solution with Substrate

Since $\bar{\omega}\bar{\tau} = \omega\tau$, the "attenuation factor"

$$\frac{\bar{A}}{\bar{E}} = \frac{A}{E} = \frac{1}{[(\omega\tau + Q\sqrt{\bar{\omega}\bar{\tau}})^2 + (1 + Q\sqrt{\bar{\omega}\bar{\tau}})^2]^{1/2}} \tag{52}$$

and the phase angle ϕ are plotted on Figures 3 and 4 for a range of Q. When Q = 0 there is no substrate, and the attenuation is of the hot wire type (i.e. eq. (38)). Notice that as soon as substrate is added to a specific film, $\bar{A}/\bar{E}$ always drops i.e. the attenuation increases. This is a vital point, since the question is often asked whether "hot films are better than hot wires". This question is wrong because it is too general. If a wire is compared with a film of the same inherent time constant τ, then the film is always worse (i.e. it attenuates more) than the hot wire. This is true for a certain frequency range only; at very high frequencies film and wire are "identical", i.e. their attenuation is of the hot wire type (6 db/octave).

The 3 db/octave film attenuation characteristic was originally mentioned by Ling (Reference 1) and Ling and Hubbard (Reference 2). A graph published in the latter Reference compares the film to the hot wire quite favorably but also improperly, because there is no accompanying mention of the inherent time constant and loss factors.

7. Film Probe Response to Specific Flow Fluctuations

7.1 Introductory

In the foregoing, the various types of flow fluctuations (i.e. velocity, density etc.) have been indiscriminately represented by the factor $\bar{E}$ in equation (49), so as to first focus attention on the frequency response issue. The next objective is to express $\bar{E}$ in terms of these fluctuations, i.e. to analyze the "forcing function" appearing on the r.h.s. of eq. (31). Specifically we want to put the forcing function into the form

$$f\theta'_e - f'(\theta-\theta_e) = \sum_i e_i \frac{v'_i}{v_i} \tag{53}$$

where v_i' is the time-dependent small fluctuation in some flow property (e.g.

velocity u' or density ρ'), v_i the corresponding mean value and e_i the corresponding sensitivity coefficient.

At the outset, it must be said that since the forcing function is identical to that of a hot wire, the answer must be identical to that supplied by Morkovin (Reference 13). Here we will therefore supply results which are the same as Morkovin's with the following differences: (a) No circuit impedance losses are considered, (b) the controllable variable is the overheat r rather than Morkovin's A'_w, (c) the resistance-temperature relation is herein assumed linear.

7.2 Fundamental Relations for Small Fluctuations

Since

$$f\vartheta'_e - f'(\vartheta-\vartheta_e) = f\,(\vartheta-\vartheta_e)\,\left[\frac{\vartheta'_e}{\vartheta-\vartheta_e} - \frac{f'}{f}\right] \tag{54}$$

we will first consider the dependence of the bracket on the fluctuations

$$T_o' \ll T_o \text{ (total temperature)} \tag{55}$$

$$u' \ll u \text{ (velocity)} \tag{56}$$

$$\rho' \ll \rho \text{ (density)} \tag{57}$$

where the r.h.s. of these inequalities, without primes, represent the mean values. Assume, with Morkovin, that the Nusselt no. N depends on Mach number M, overheat $r = (R - R_e)/R_e$ and Reynolds number based on stagnation conditions:

$$Re \equiv \frac{\rho u d}{\mu_o} \tag{58}$$

so that

$$N = N\ (M,\ Re,\ r) \tag{59}$$

Small fractional fluctuations (denoted by either a differential or a prime) then are:

$$\frac{dN}{N} = \frac{N'}{N} = \frac{1}{N}\ \left(\frac{\partial N}{\partial Re}\ dRe + \frac{\partial N}{\partial M}\ dM + \frac{\partial N}{\partial r}\ dr\right) = N_{Re}\ \frac{Re'}{Re} + N_M\ \frac{M'}{M} + N_r\ \frac{r'}{r} \tag{60}$$

where, for brevity, we symbolize for any two quantities P and Q:

$$P_Q = \frac{Q}{P}\ \frac{\partial P}{\partial Q} \tag{61}$$

Now assume the viscosity-temperature relation

$$\mu \sim T^n,\ \mu_o \sim T_o^{\,n} \tag{62}$$

which gives

$$\frac{Re'}{Re} = \frac{du}{u} + \frac{d\rho}{\rho} - n\,\frac{dT_o}{T_o} = \frac{u'}{u} + \frac{\rho'}{\rho} - n\,\frac{T_o'}{T_o} \tag{63}$$

For dM/M we utilize Morkovin's expression (eq. (25) of Ref. 13):

$$\frac{dM}{M} = \frac{M'}{M} = \delta\ \left(\frac{u'}{u} - \frac{1}{2}\ \frac{T_o'}{T_o}\right); \quad \delta = 1 + \frac{\gamma - 1}{2}\ M^2 \tag{64}$$

Since, upon assuming a reference temperature $T_r = 0$,

$$R = R_r\ (1 + \alpha T)\ ,\ R_e = R_r\ (1 + \alpha T_e) \tag{65}$$

and

$$r = \frac{R - R_e}{R_e} = \frac{\alpha (T - T_e)}{1 + \alpha T_e} \tag{66}$$

we also get

$$\frac{dr}{r} = \frac{r'}{r} = \frac{T'}{T - T_e} - \frac{T_e(1+\alpha T)}{(T-T_e)(1+\alpha T_e)} \quad \frac{T'_e}{T_e} \tag{67}$$

and since $T_e = \eta T_o$:

$$\frac{T_e'}{T_e} = \frac{\eta'}{\eta} + \frac{T_o'}{T_o} \tag{68}$$

The recovery factor dependence for air is known to be

$$\eta = \eta(M, Re), \frac{\eta'}{\eta} = \frac{d\eta}{\eta} = \eta_M \frac{M'}{M} + \eta_{Re} \frac{Re'}{Re} \tag{69}$$

Combining equations (63), (64), (66), (67) and (68),

$$\frac{r'}{r} = \frac{T'}{T-T_e} - \frac{T_e(1+\alpha T)}{(T-T_e)(1+\alpha T_e)} \Big[\frac{T_o'}{T_o} + \eta_M \, \delta\Big(\frac{u'}{u} - \frac{1}{2}\frac{T_o'}{T_o}\Big)$$

$$+ \eta_{RE} \Big(\frac{u'}{u} + \frac{\rho'}{\rho} - n\frac{T_o'}{T_o}\Big)\Big] \tag{70}$$

Equations (63), (64) and (70) can now be substituted into (60):

$$\frac{N'}{N} = \frac{dN}{N} = N_{Re}\Big(\frac{u'}{u} + \frac{\rho'}{\rho} - n\frac{T_o'}{T_o}\Big) + N_M \, \delta\Big(\frac{u'}{u} - \frac{1}{2}\frac{T_o'}{T_o}\Big) + \frac{N_r T'}{T-T_e}$$

$$- \frac{N_r T_e(1+\alpha T)}{(T-T_e)(1+\alpha T_e)} \Big[\frac{T_o'}{T_o} + \eta_M \, \delta\Big(\frac{u'}{u} - \frac{1}{2}\frac{T_o'}{T_o}\Big) + \eta_{Re}\Big(\frac{u'}{u} + \frac{\rho}{\rho} - n\frac{T_o'}{T_o}\Big)\Big] \tag{71}$$

In addition, we need the fluctuation in the thermal conductivity, which we can consider to be

$$k_o \sim T_o^m \tag{72}$$

so that

$$\frac{k_o'}{k_o} = m\,\frac{T_o'}{T_o} \tag{73}$$

which concludes the collection of fundamental relations needed.

7.3 Development of the Forcing Function

We now develop the bracket of eq. (54) with the aid of the definitions of f, ϑ, ϑ_e, (eqs. (12), (13) and (15)) and of relations (63), (64) etc.:

$$\frac{\vartheta_e'}{\vartheta-\vartheta_e} - \frac{f'}{f} = \frac{\vartheta_e}{\vartheta-\vartheta_e}\;\frac{\vartheta'_e}{\vartheta_e} - \frac{f'}{f} = \frac{T_e}{T-T_e}\;\frac{T_e'}{T_e} - \left(\frac{k_o'}{k_o} + \frac{N'}{N}\right) \tag{74}$$

Expanding the fluctuations of k_o, T_e and N into their constituents,

$$\frac{\vartheta'_e}{\vartheta-\vartheta_e} - \frac{f'}{f} = \frac{T_e}{T-T_e}\left[\frac{T_o'}{T_o} + \eta_M\,\delta\left(\frac{u'}{u} - \frac{1}{2}\,\frac{T_o'}{T_o}\right) + \eta_{Re}\left(\frac{u'}{u} + \frac{\rho'}{\rho} - n\,\frac{T_o'}{T_o}\right)\right]$$

$$-m\,\frac{T_o'}{T_o} - \left[N_{Re}\left(\frac{u'}{u} + \frac{\rho'}{\rho} - n\,\frac{T_o'}{T_o}\right) + N_M\,\delta\left(\frac{u'}{u} - \frac{1}{2}\,\frac{T_o'}{T_o}\right) + \frac{N_r T'}{T-T_e} - \frac{N_r T_e(1+\alpha T)}{(T-T_e)(1+\alpha T_e)}\right.$$

$$\left.\left\{\frac{T_o'}{T_o} + \eta_M\,\delta\left(\frac{u'}{u} - \frac{1}{2}\,\frac{T_o'}{T_o}\right) + \eta_{Re}\left(\frac{u'}{u} + \frac{\rho'}{\rho} - n\,\frac{T_o'}{T_o}\right)\right\}\right] =$$

$$\frac{T_o'}{T_o}\left[\frac{T_e}{T-T_e}\left(1+\frac{N_r(1+\alpha T)}{1+\alpha T_e}\right)\left(1-n\eta_{Re} - \frac{\delta}{2}\,\eta_M\right) + nN_{Re} + \frac{\delta}{2}\,N_M - m\right]$$

$$+ \frac{u'}{u} \left[\frac{T_e}{T-T_e} \left(1+ \frac{N_r(1+\alpha T)}{1+\alpha T_e}\right) (\delta\eta_M + \eta_{Re}) - N_{Re} - \delta N_M\right]$$

$$+ \frac{\rho'}{\rho} \left[\frac{T_e}{T-T_e} \left(1 + \frac{N_r(1+\alpha T)}{1+\alpha T_e}\right) \eta_{Re} - N_{Re}\right] - \frac{N_r T'}{T-T_e} \tag{75}$$

The last term in this equation, containing T′, will be omitted from further considerations since it has already been carried to the l.h.s. of eq. (34). (See discussion in Section 6.1). The complete forcing function as given by (54) can now be found by noting that the factor

$$f\ (\vartheta-\vartheta_e) = \frac{A_f k_o N w}{\epsilon B}\ \alpha(T-T_e) = \frac{A_f k_o N w}{\epsilon B}\ r\ \frac{R_e}{R_r} \tag{76}$$

To get the r.h.s. of eq. (34) we now multiply this factor with eq. (75):

$$f(\vartheta-\vartheta_e)\left(\frac{\vartheta_e'}{\vartheta-\vartheta_e} - \frac{f'}{f}\right) = \frac{A_f k_o N w}{\epsilon B}\ \frac{R_e}{R_r}\ \left(\frac{T_o'}{T_o}\ [G\ (1-n\eta_{Re} - \frac{\delta}{2}\ \eta_M)\right.$$

$$+r(nN_{Re} + \frac{\delta}{2}\ N_M - m)] + \frac{u'}{u}[G\ (\delta\eta_M + \eta_{Re}) - r(N_{Re} + \delta N_M)]$$

$$\left. + \frac{\rho'}{\rho}\ [G\eta_{Re} - rN_{Re}]\right) \tag{77}$$

where the T′-containing term of eq. (75) is missing by design, as noted, where

$$G = \frac{\alpha T_e}{1+\alpha T_e}\ \left(1 + \frac{N_r(1+\alpha T)}{1+\alpha T_e}\right) = \frac{\alpha T_e}{1+\alpha T_e}\ [1+N_r(r+1)] \tag{78}$$

and where η_M, η_{Re}, N_{Re} etc. have been defined via eq. (61) and δ via eq. (64).

Eq. (77) can be furthermore re-written if we utilize the definition of the non-dimensional film time constant $\bar{\tau}$ from eq. (39) as follows:

$$f(\vartheta-\vartheta_e)\left(\frac{\vartheta_e'}{\vartheta-\vartheta_e}-\frac{f'}{f}\right)=\frac{A_f k_o N w}{\epsilon B}\,\frac{R_e}{R_r}\,\{\ldots\ldots\}=$$

$$\frac{A_f k_o N w}{\epsilon B}\,\frac{R_e}{R_r}\,\frac{N_r+\frac{R_e}{R}}{N+\frac{R_e}{R}}\,\{\ldots\}=\frac{1}{\bar{\tau}}\,\frac{R}{R_r}\,[N_r\frac{R}{R_e}+1]^{-1}\,\{\ldots\} \tag{79}$$

and eq. (77) can then be rewritten with the factor outside the curly bracket replaced by $(1/\bar{\tau})(R/R_r)$ and with G and r replaced by

$$\bar{G}=\frac{\alpha T_e}{1+\alpha T_e} \tag{80}$$

$$\bar{r}=\frac{r}{(r+1)\ N_r+1} \tag{81}$$

8. <u>Final Solution for the Film Temperature Perturbation</u>

The fluctuations To'/To, u'/u, and ρ'/ρ in eq. (77) have been assumed to be functions of the time. If each is representable by a Fourier series in $\sin(\omega t)$ and if we concentrate on the Fourier component at a specific frequency ω, we can write

$$\frac{T_o'}{T_o}=\hat{T}_o\sin\omega t,\quad \frac{u'}{u}=\hat{u}\sin\omega t,\quad \frac{\rho'}{\rho}=\hat{\rho}\sin\omega t \tag{82}$$

where $(\hat{\ })$ stands for the non-dimensional amplitude (e.g. $\hat{T}_o$ is the small fluctuation amplitude in T_o, divided by the mean value of T_o). In view of eq. (82), we can then re-write eq. (77) as follows:

$$f\ (\vartheta-\vartheta_e)\left(\frac{\vartheta'_e}{\vartheta-\vartheta_e}-\frac{f'}{f}\right)=\frac{1}{\bar{\tau}}\,\frac{R}{R_r}\,(\hat{T}_o\,[\bar{G}(1-n\eta_{Re}-\frac{\delta}{2}\eta_M)$$

$$+\ \bar{r}\ (nN_{Re}+\frac{\delta}{2}N_M-m)]+\hat{u}\,[\bar{G}\ (\delta_{\eta M}+\eta_{Re})-\bar{r}\ (N_{Re}+\delta N_M)] \tag{83}$$

$$+\ \hat{\rho}[\bar{G}\eta_{Re}-\bar{r}N_{Re}])$$

But we note that, since $\bar{\omega}\bar{\tau}=\omega\tau$, this forcing function has exactly the same form

as that on the r.h.s. of eq. (34), with

$$\bar{E} = \frac{R}{R_r}(\ldots.) \tag{84}$$

Therefore, since the solution for eq. (34) is given by eq. (49), by analogy we see that in the present instance, the solution for θ' is

$$\alpha T' \equiv \theta'(\bar{t}) = \frac{R}{R_r}\ \frac{\sin(\bar{\omega}\bar{t}-\phi)}{[(\bar{\omega}\bar{\tau} + Q\sqrt{\bar{\omega}\bar{\tau}})^2 + (1 + Q\sqrt{\bar{\omega}\bar{\tau}})^2]^{1/2}}\ \{\hat{T}_o\ [\bar{G}\ (1-n\eta_{Re}$$

$$- \frac{\delta}{2}\eta_M) + \bar{r}\ (nN_{Re} + \frac{\delta}{2}N_M - m)] + \hat{u}\ [\bar{G}\ (\delta\eta_M + \eta_{Re})$$

$$- \bar{r}\ (N_{Re} + \delta N_M)] + \hat{\rho}\ [\bar{G}\eta_{Re} - \bar{r}N_{Re}]\} \tag{85}$$

and with ϕ and Q remaining as given in eqs. (49) and (50) and τ or $\bar{\tau}$ as given in (39) and (40). (Note again that $\bar{\omega}\bar{\tau} = \omega\tau$).

9. Film Voltage Fluctuations

The fluctuation e′ in the film voltage can be obtained directly from eq. (85), since it is related to the mean film voltage e as follows. Since for constant current

$$e + e' = i(R + R') = iR_r\ [1 + \alpha\ (T + T')] \tag{86}$$

then $$e' = i\alpha R_r T'\ ,\ e \equiv iR \tag{87}$$

So that by multiplying both sides of (85) by iR_r we get

$$e' = i\alpha R_r T' = \frac{iR\sin(\omega t-\phi)}{[\quad]^{1/2}}\{\ldots.\} = \frac{e\sin(\omega t-\phi)}{[\quad]^{1/2}}\{\ldots.\} \tag{88}$$

10. Practical Aspects of the use of the Film Anemometer

10.1 Mean-Flow Measurement Routine

The universal mean heat-transfer characteristics of films (i.e. the N (Re, M, r) and η (Re, M) dependences) are of fundamental interest only if they are composed from, and afterward applied to, films of accurately known geometry. Such accuracy is impossible for home-made film probes*, and universal characteristic curves replacing calibration curves will therefore not be available. However, a specific film probe can be calibrated and then used for a measurement without the need of knowing its exact geometry. In all cases, a measurement of the Nusselt number is needed. The procedure for this measurement is generally indicated in Reference 8.

According to eq. (27),

$$\bar{N} = N + \frac{k_s}{k_o}\,\frac{h}{L} \tag{89}$$

or

$$w\bar{N} = wN + \frac{k_s}{k_o}\,\frac{hw}{L} \tag{90}$$

According to Reference 8, what is usually measured (and plotted vs Re′ or Re_h) is the $w\bar{N}$ on the l.h.s. of eq. (90). (In that Reference the symbol $\bar{N}$ stood for $w\bar{N}$ as defined here). If w is known, N can next be obtained from eq. (89) provided that (k_s/k_o) (h/L) can be computed, which is improbable because of uncertainties in L. However, (k_s/k_o) (h/L) does not depend on Re′, and can be obtained from the intercept at Re′ = 0 of a plot of $\bar{N}$ vs Re′. Thus a plot of N vs Re′ is obtainable.

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* By contrast hot wire probes, even of the home-made variety, enjoy the precision established by industrial standards for the wire material, e.g. its diameter and resistivity.

If w is unknown eq. (90) is operative, the intercept just mentioned will signify $(k_s/k_o)\,(hw/L)$ rather than $(k_s/k_o)\,(h/L)$, and the final result will be a value of $(w\bar{N})$ for each Re' or Re_h.

When the final objective is to deduce Re' in an unknown flow by measuring the Nusselt number of a specific film, it is adequate to use a film calibrated as $(w\bar{N})$ vs Re', provided that the total temperature for calibration and measurement is the same (this will ensure that $(k_s/k_o)\,(wh/L)$ is the same). If the two total temperatures differ by a known amount, changes in the conduction loss can still be estimated based on changes in k_o.

For fluctuation measurements, it has been seen in Section 7.2 that the Nusselt no. enters the sensitivities via its logarithmic derivatives, e.g.

$$N_{Re} = \frac{Re}{N}\,\frac{\partial N}{\partial Re} = \frac{Re'}{w\bar{N}}\,\frac{\partial w\bar{N}}{\partial Re'} = \frac{\partial w\bar{N}}{\partial Re'}\,\frac{Re'}{w\bar{N} - \frac{k_s}{k_o}\frac{wh}{L}} \tag{91}$$

which can be computed from a knowledge of the $w\bar{N}(Re')$ curve and the determination of the conduction loss from that curve.

10.2 The Unsteady Loss Factor Q Due to the Substrate

In eq. (49), it was shown that the film probe attenuates unsteady signals in a way dependent on two parameters: the inherent film time constant τ and a loss factor Q. For finite Q the film response is quite complicated (see Fig. 3) and one cannot use electronic compensation circuits employed for hot wires. Therefore some knowledge of the magnitude of Q is needed for typical films.

The loss factor Q for zero overheat (unheated film) is given by eq. (51) which can be re-written in the following way:

$$Q \equiv \frac{k_s A_f}{\epsilon B}^{3/2} \sqrt{\frac{\bar{\tau}}{2}} = \left[\frac{1}{2N} \left(\frac{k_s}{k_e}\right) \left(\frac{\rho_s C_s}{\rho_f C_f}\right) \left(\frac{h}{d}\right)\right]^{1/2} \tag{92}$$

To make a typical calculation, assume that the substrate (subscript s) is quartz glass with the following properties (which are taken from Reference 14 for 0° C, with the page numbers noted).

- Quartz thermal conductivity (p. 1867): $k_s = 0.023 \ \frac{\text{cal}}{\text{cm.sec.deg C}}$ (93)

- Quartz density (p. 1689): $\rho_s = 2.65 \ \frac{\text{gm}}{\text{cm}^3}$ (94)

- Quartz heat capacity (p. 1790): $c_s = 0.235 \ \frac{\text{cal}}{\text{gm.deg C}}$ (95)

The k_e is the thermal conductivity of stagnant air at 0° C on the surface of the unheated film:

$$k_e = 5.68 \times 10^{-5} \ \frac{\text{cal}}{\text{cm.sec deg C}} \tag{96}$$

The film material is taken to be platinum, and although ultra-thin films usually have properties differing from those of bulk materials (Reference 6), bulk-material platinum properties will be assumed, again at 0 deg. C.:

- Platinum density (p. 461): $\rho_f = 21.45 \text{ gm/cm}^3$ (97)
- Platinum heat capacity (p. 1773): $c_f = 0.03162 \ \frac{\text{cal}}{\text{gm deg C}}$ (98)

The film dimensions will be assumed to be

$$h = 0.04 \text{ cm} \tag{99}$$

$$w = 0.1 \text{ cm}$$

As Ling points out (Reference 1), the film thickness d can be computed from its resistance, using a resistivity of platinum (at 0° C) of

$$r_R = 10.96 \text{ microohm.cm}$$

We will assume as typical a film resistance R = 20 ohms at 0°C; then

$$d = \frac{r_R W}{hR} = 1.3 \times 10^{-6} \text{ cm} \tag{100}$$

Finally, we will adopt values of the Nusselt number N taken from measurements of the compound Nusselt number $\bar{N}$ already made at MSU (Reference 8); for example,

$$\bar{N} = 30 \tag{101}$$

If the values (93) - (101) are substituted into (92), one obtains

$$Q = 436 \tag{102}$$

This loss factor means that the substrate completely dominates the film performance; see Figure 3.

10.3 The Inherent Time Constant of the Film

To complete the computation of attenuation, the value of τ (the inherent time constant) is needed. This can be obtained from eq. (40). Here we compute the unheated inherent τ (for r = 0), using the numerical values given by (93) - (101) with added assumption that $N_r = 0$:

$$\tau = \frac{\epsilon}{k_o N} = \frac{\rho_f c_f h d}{k_o N} = 21 \times 10^{-6} \text{ sec} \tag{103}$$

Figure 5 shows that with the substrate in place the attenuation is much larger than without it. This film attenuates more than even a large-time-constant hot-wire, e.g. one of 200 microseconds.

10.4 Measurement of the Response

Because of the sensitivity of the frequency response to the local flow conditions (via the k_eN or k_oN parameters), any response measurements needed to verify eq. (52) must be made "in situ". The most convenient such measurement is to record the attenuation caused by the film to an impressed time-dependent (e.g. periodic) heating current. The issue arising here is whether the attenuation is the same for such a variable current as it would be for variable flow properties. Assurance to this effect is mentioned by Kovasznay for hot-wires (Reference 7) and by Ling for films (Reference 1). Proof of these assertions will now be given.

10.4.1 Checking the Response When the Substrate is Absent

First we investigate the response of the film, without substrate, immersed in a quiescent flow:

$$T_o' = u' = \rho' = 0$$

So that the forcing function on the r.h.s. of eq. (33) is simply*

$$f\theta'_e - f'(\theta-\theta_e) = -fN_r\theta' \tag{104}$$

when the heating current is

$$i + i'(t),\ i \text{ steady},\ i' << i \tag{105}$$

Assume, furthermore, the harmonic input

$$i'/i = I \sin wt,\ I << 1 \text{ (a pure number)} \tag{106}$$

Inserting (104) and (105) into the general equation (17) for small perturbations, and considering the substrate absent, we get

*For example, consider the r.h.s. of eq. (75) when $To' = u' = \rho' = 0$.

$$\frac{d\vartheta'}{d\bar{t}} = \frac{\alpha R_r A_f}{\epsilon B}\,(i^2 + 2ii')\,(1 + \vartheta + \vartheta') - f(\vartheta + \vartheta' - \vartheta_e) - fN_r\vartheta' \tag{107}$$

which in the steady state becomes

$$B\,(1+\vartheta) - f(\vartheta-\vartheta_e) = 0 \tag{108}$$

giving eq. (24) as the final steady-state result. Next, the unsteady equation is obtained by combining (107) and (108):

$$\frac{d\vartheta'}{d\bar{t}} + \vartheta'[f(1 + N_r)-B] = 2B\,(1+\vartheta)\,\frac{i'}{i} \tag{109}$$

The r.h.s. of (109) is a forcing function replacing the forcing function on the r.h.s. of (31), when $k_s = 0$. Equations (31) and (109) are identical otherwise, and therefore their time constants are the same, proving Kovasznay's statement. Therefore the film time constant found by imposing a current fluctuation is the same as that prevailing under flow fluctuations in the absence of substrate.

It is useful to proceed with the solution of (109) which, according to eq. (38):

$$\alpha T' \equiv \vartheta'(\bar{t}) = [2B(1+\vartheta)\,\bar{\tau}\,I]\;\frac{\sin(\bar{\omega}\bar{t} - \phi)}{(1+\bar{\omega}^2\bar{\tau}^2)^{1/2}} \tag{110}$$

with $\bar{\tau}$ from (39). With the aid of eq. (40) the bracketed factor in (110) is

$$2B(1+\vartheta)\bar{\tau}I = \frac{2Ii^2\alpha R^2}{k_o N R_e} \tag{111}$$

Now note that since the total voltage drop across the film is $e + e'(t) = iR + e'(t)$, we have

$$e' = d(iR) = Ri' + i\alpha R_r T' \tag{112}$$

and therefore, utilizing eq. (110),

$$\frac{e'}{e} = I\sin\omega t + \frac{R_r}{R}\ \frac{2Ii^2\alpha R^2}{k_o N R_e}\ \frac{\sin(\omega t-\phi)}{(1+\tau^2\omega^2)^{1/2}} \tag{113}$$

$$\frac{e'}{e} = I\left(\sin\omega t + 2r\frac{\sin(\omega t-\phi)}{(1+\tau^2\omega^2)^{1/2}}\right) \tag{114}$$

In many practical circuits the $I\sin\omega t$ component of e'/e is electronically cancelled ("resistive component cancellation"), giving a "net" film voltage signal

$$\frac{e'_n}{e} = 2rI\frac{\sin(\omega t-\phi)}{(1+\tau^2\omega^2)^{1/2}} \tag{115}$$

This equation can be used for one of the many possible ways of measuring τ. For example, the mean square (m.s.) of e'_n at a certain frequency ω can be compared with its equivalent at $\omega = 0$; then

$$\tau = \frac{1}{\omega}\left[\frac{e'_{n\ m.s.}(0)}{e'_{n\ m.s.}(\omega)} - 1\right]^{1/2} \tag{116}$$

10.4.2 Checking the Response With the Substrate Present

It remains to consider the effect of a current fluctuation in the presence of the substrate. The development follows closely the lines of the previous section. We begin again by considering the effect of equations (104) and (105) on eq. (17):

$$\frac{d\theta'}{dt} = \frac{\alpha R_r A_f i^2}{\epsilon B}(1+\theta) + B\theta' + 2ii'\frac{\alpha R_r A_f}{\epsilon B}(1+\theta) - f(\theta-\theta_e)$$

$$-f\theta' + \frac{k_s A_f}{\epsilon B}^{3/2}\left[\left(\frac{\partial\theta_s}{\partial X}\right)_{X=0} + \left(\frac{\partial\theta'_s}{\partial X}\right)_{X=0}\right] - \theta' f N_r \tag{117}$$

In the steady state this reduces to eq. (18) once again, with the solution given by (24). When (18) and (117) are combined, the unsteady equation is

$$\frac{\partial\theta'}{\partial t} + \theta' \, [f(1+N_r)-B] - k_s \, \frac{A_f^{3/2}}{\epsilon B} \left(\frac{\partial\theta'_s}{\partial X}\right)_{X=0} = 2 \, \frac{i'}{i} \, B \, (1+\theta') \tag{118}$$

This equation is identical to eq. (31) except that the forcing function is again now driven by the fluctuating current. Therefore the time constants of the systems described by eqs. (31) and (118) are the same, proving Ling's proposition.

We conclude with the solution of (118). If we take

$$\theta' = \bar{A} \sin (\omega t-\phi) \tag{119}$$

$$\frac{i'}{i} = I\sin\omega t \tag{120}$$

we again obtain eq. (47), giving the following result:

$$\begin{aligned} &\bar{A}\bar{\omega} \cos(\omega t-\phi) + \frac{\bar{A}}{\bar{\tau}} \sin(\omega t - \phi) + \frac{k_s A_f^{3/2}}{\epsilon B} \bar{A} \sqrt{\frac{\bar{\omega}}{2}} [\sin(\omega t-\phi) + \cos(\omega t-\phi)] \\ &\quad = 2IB \, (1+\bar{\theta}) \sin\omega t \end{aligned} \tag{121}$$

The amplitude $\bar{A}$ and phase angle ϕ are found once more by zeroing the coefficients of $\cos\omega t$ and $\sin\omega t$, and by utilizing eqs. (111) and (112) we find that in this case the counterpart of eq. (114) is:

$$\frac{e'}{e} = I\left(\sin\omega t + 2r \, \frac{\sin(\omega t-\phi)}{[(\omega\tau + Q\sqrt{\omega\tau})^2 + (1 + Q\sqrt{\omega\tau})^2]^{1/2}}\right) \tag{122}$$

The net r.m.s. e'_n of the voltage across the film (i.e. when the resistive component $I\sin\omega t$ is cancelled) is

$$\frac{e'_{nrms}}{e} = \frac{Ir\sqrt{2}}{[\ldots.]^{1/2}} \tag{123}$$

The objective, which is the knowledge of the bracket in the denominator of (123), can now be attained by measuring all other properties in eq. (123) at any moment of the film operation in a given flow.

Finally, it may appear somewhat odd that the voltage fluctuations given in eqs. (115) and (122) depend on the overheat r, especially since this would make the time-constant measurement difficult at low r. The explanation is that the impressed current fluctuations i' are assumed to be a small fraction of the steady-state i. Therefore, as i (and hence r) tends to zero so does the assumed i' and hence also the voltage fluctuations e'_n. It would be valuable to analyze the case where i' is small but nevertheless much larger than any steady current. This would represent the "a.c.-heated" film.

10.5 Electronic Compensation of the Hot Film

The sole reason for the usefulness of the hot-wire at high speeds is the simplicity of its attenuation characteristic, given by eq. (38):

$$\frac{e_{OUT}}{e_{in}} = \frac{1}{[1 + \omega^2\tau^2]^{1/2}} , \quad \lim_{\omega\tau \to \infty} \frac{e_{OUT}}{e_{in}} = \frac{1}{\omega} \tag{124}$$

for which a compensating amplifier with a gain increasing as ω can be built.

To equip the film probe with a similar compensator, one would hope for similar simplicity in its attenuation. Pessimism in that direction prevails in the earlier hot-film analyses, which replace eq. (124) by complicated functions of ω, due mainly to boundary conditions different from those used here. For example, in References 3, 4, and 5, one finds attenuation functions with minima, maxima, regions of insensitivity to ω, etc. We shall now show that the present results simplify this function considerably, with hopes of building its electronic inverse for compensation purposes.

The attenuation found herein is given by eq. (49):

$$\frac{e_{OUT}}{e_{in}} = \frac{1}{[(\omega\tau + Q\sqrt{\omega\tau})^2 + (1 + Q\sqrt{\omega\tau})^2]^{1/2}} \tag{125}$$

This function is not necessarily a smooth one in ω since it goes as the inverse square root of frequency up to some ω, and as the inverse frequency thereafter. However, if

$$Q >> 1 \tag{126}$$

$$Q >> \sqrt{\omega\tau} \tag{127}$$

then in our present case eq. (125) gives

$$\frac{e_{OUT}}{e_{IN}} = \frac{1}{\sqrt{\omega}} \tag{128}$$

In Section 10.2 we quote estimates of Q = 436, τ = 20 microseconds. For these values the criteria (126) and (127) are easily fulfilled to one megahertz and beyond, making very attractive the consideration of a compensator for film probes, which has a gain proportional to $\sqrt{\omega}$.

11. Conclusions

The main conclusions drawn from this analysis of constant-current film anemometers are as follows:

1) In contrast to hot-wire probes, which attenuate the input as the simple inverse of the frequency at high-frequencies, the film probe attenuation is quite complex. The film frequency response can not be described by a single time constant; rather, this response depends on two parameters: the inherent film time constant and a loss factor dependent on the substrate. For small loss factors (of order unity) the film response is almost identical to the wire response. However, the loss factor for actual probes is estimated to be of order

100 or higher. In such cases the film attenuates more than the wire at low frequencies, although its response falls off more gradually, at a 3 db/octave rate at intermediate-to-high frequencies. At extremely high frequencies, probably high enough to be of little interest, all films resemble the wires once again as regards frequency response.

Two practical results are stressed. First, a well-constructed film with small inherent time constant should "see" high frequency signals quite easily. Second, since the ultra-high frequencies are of little concern, the film response of 3 db/octave drop may be easily compensatable by an electronic circuit.

2) The inherent film time constant resembles functionally that of the wire. In contrast to previous analyses, the present theory shows that this time constant (for the wire as well) should include a term related to the dependence of the Nusselt number on the overheat.

3) The phase lag angle between input and output is identical to that of hot wires for a loss factor of zero, and constant at 45 degrees independent of frequency when the loss factor is infinite.

4) The sensitivity coefficients of the film to flow fluctuations (e.g. in the total temperature, velocity etc.) are the same as that for the hot wire. Forms given here for these coefficients alternative to those in the classic hot-wire literature are presented mainly for convenience.

5) The steady-state Nusselt number measured with the film by the classical overheat method is a "compound" one which includes conductive losses analogous to the "end corrections" for hot wires. However, whereas these wire end corrections are numerically inferior to the actual Nusselt no. measured with the wire, in the case of the film they can be substantially higher than the pure convective Nusselt number.

12. Recommendations For Further Work

In order to embody the findings of this report into a practical film

anemometer system capable of measuring mean and turbulent flows at high speeds, the following extensions of the work done to date are recommended:

1) The derivatives η_M, η_{Re}, N_M etc. appearing in the operational formulas (83), (85) etc. must be evaluated by measurements of the film Nusselt number and recovery factor dependence on Mach number, Reynolds number and overheat. The dependence of the recovery factor on Mach number is judged as especially critical.

2) Once these quantities are known over a range of the flow conditions, the sensitivity of the film probe to flow fluctuations (e.g. velocity or density fluctuations) must be calculated for the same range of conditions.

3) The calculated 3 db/octave film response must be verified experimentally. Compensating amplifier circuits must be built with such a 3db/octave gain, to replace present electronics which commonly amplify at the 6 db/octave gain characteristic of hot-wire anemometers.

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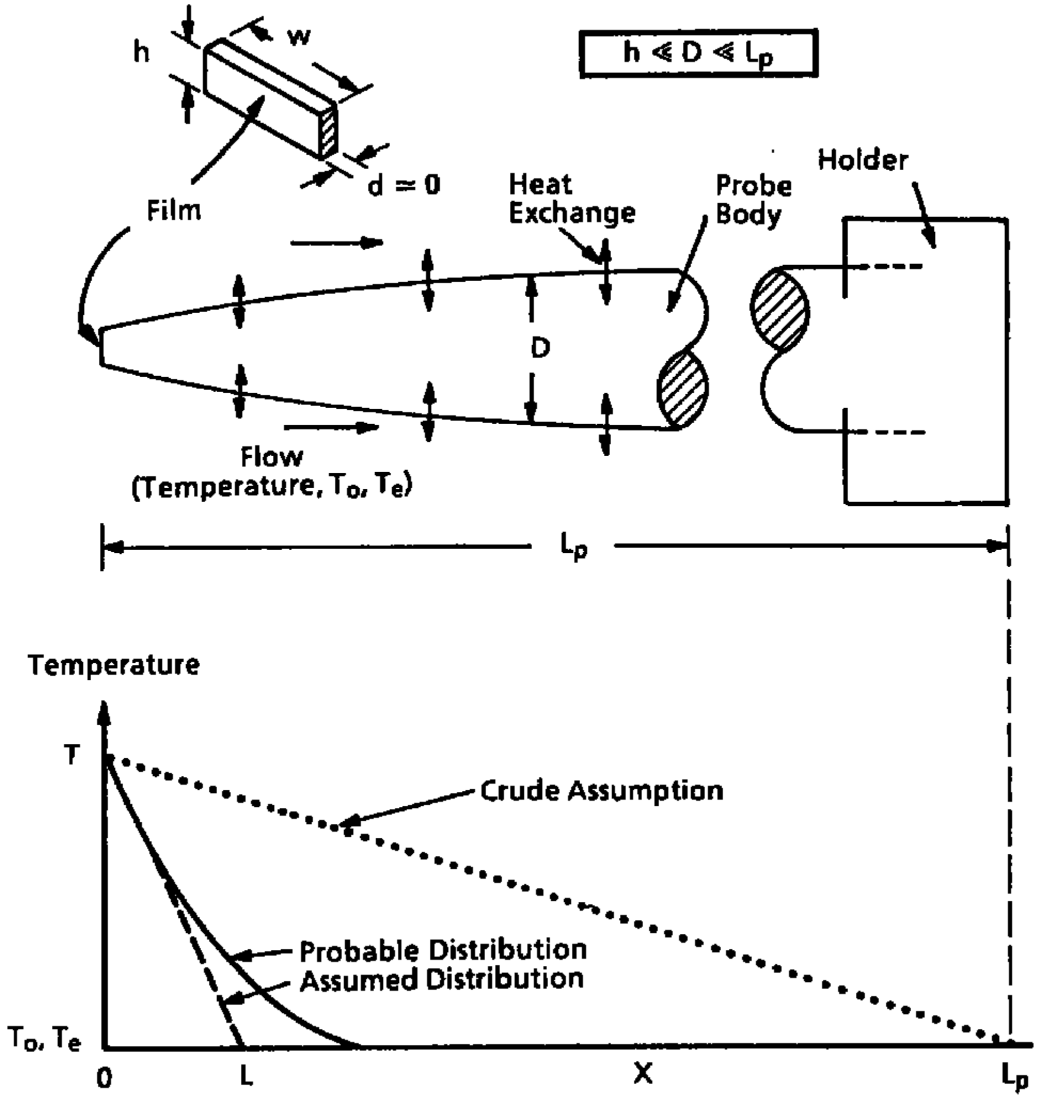

Figure 1. Typical hot-film anemometer probe configuration (above), with a general view of the temperature distribution in the probe body (below).

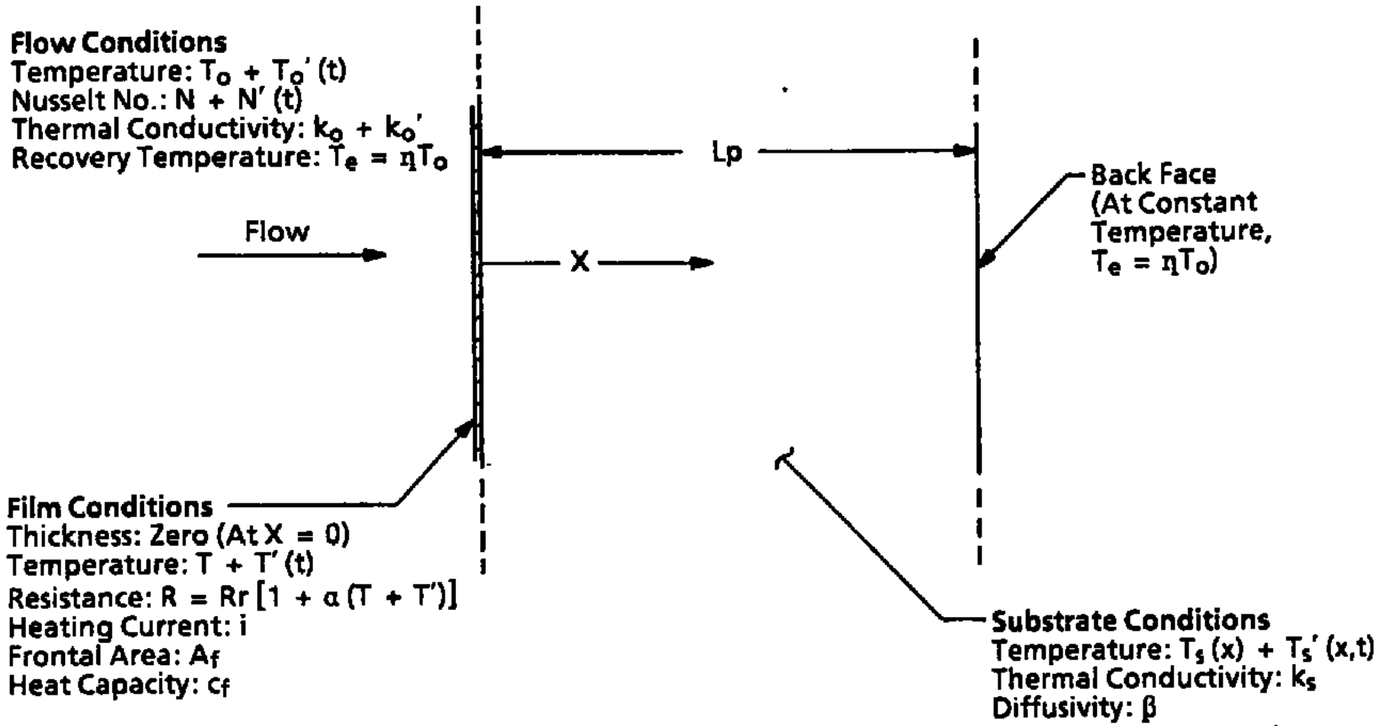

Figure 2. The physical model with definitions and nomenclature.

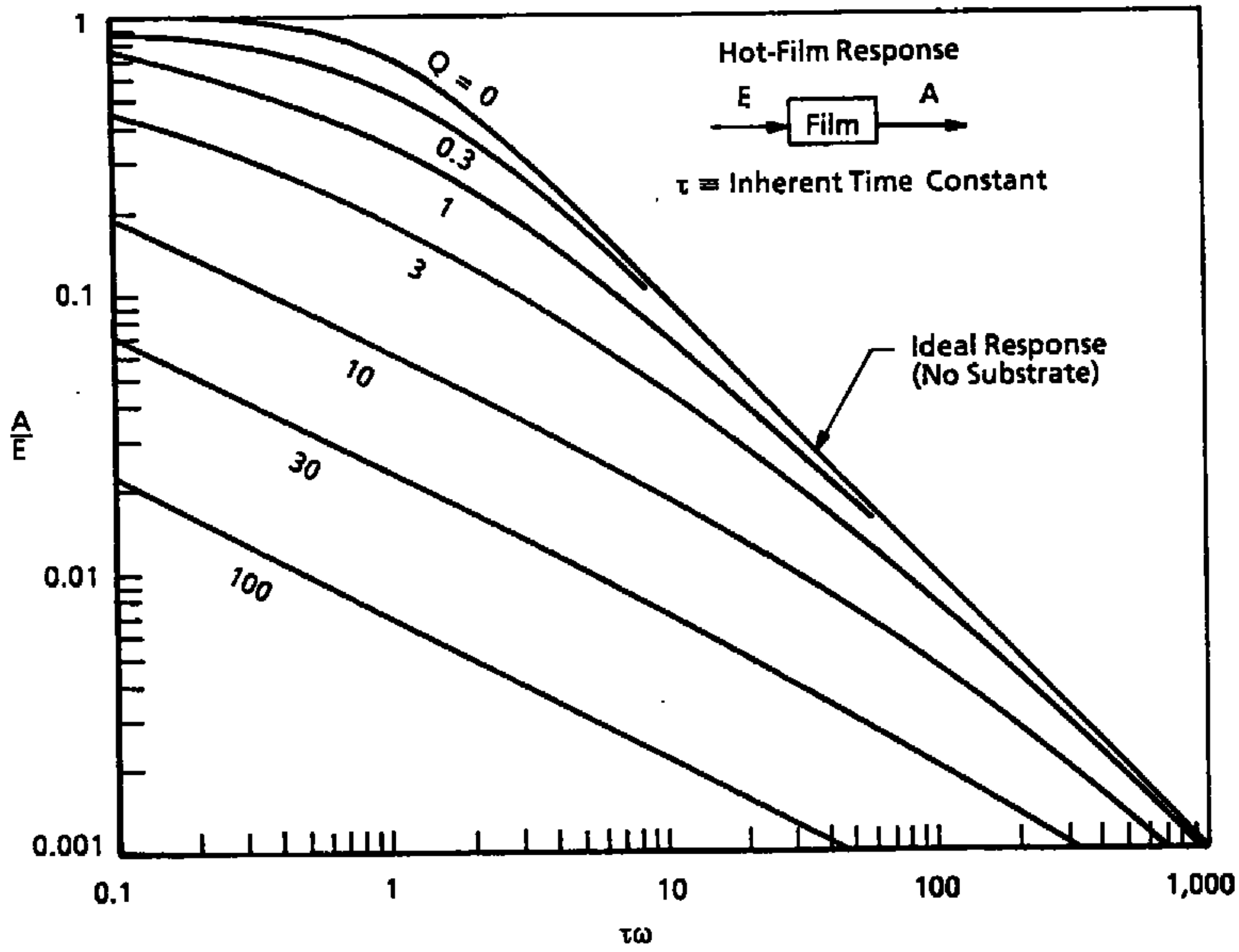

Figure 3. Attenuation dependence on frequency and substrate parameter Q.

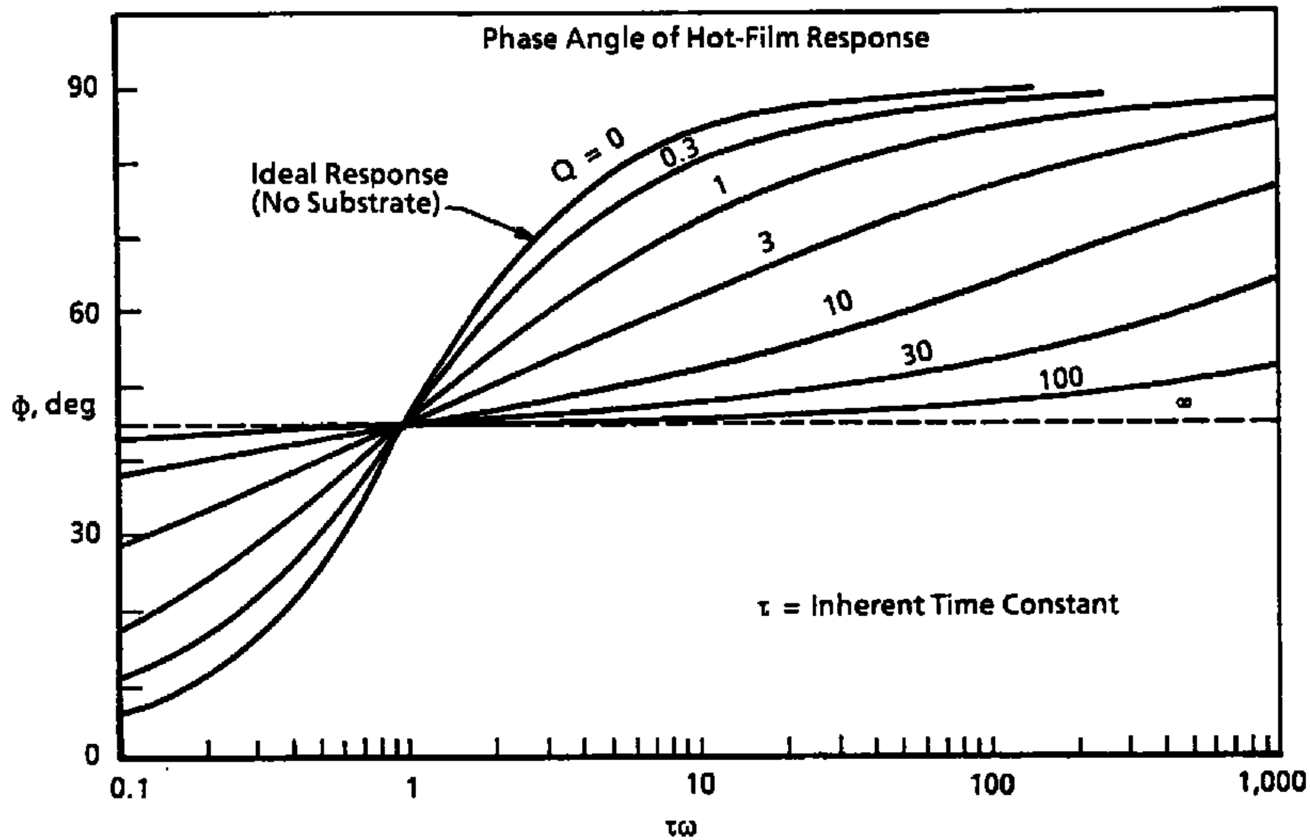

Figure 4. Phase angle dependence on frequency and substrate parameter Q.

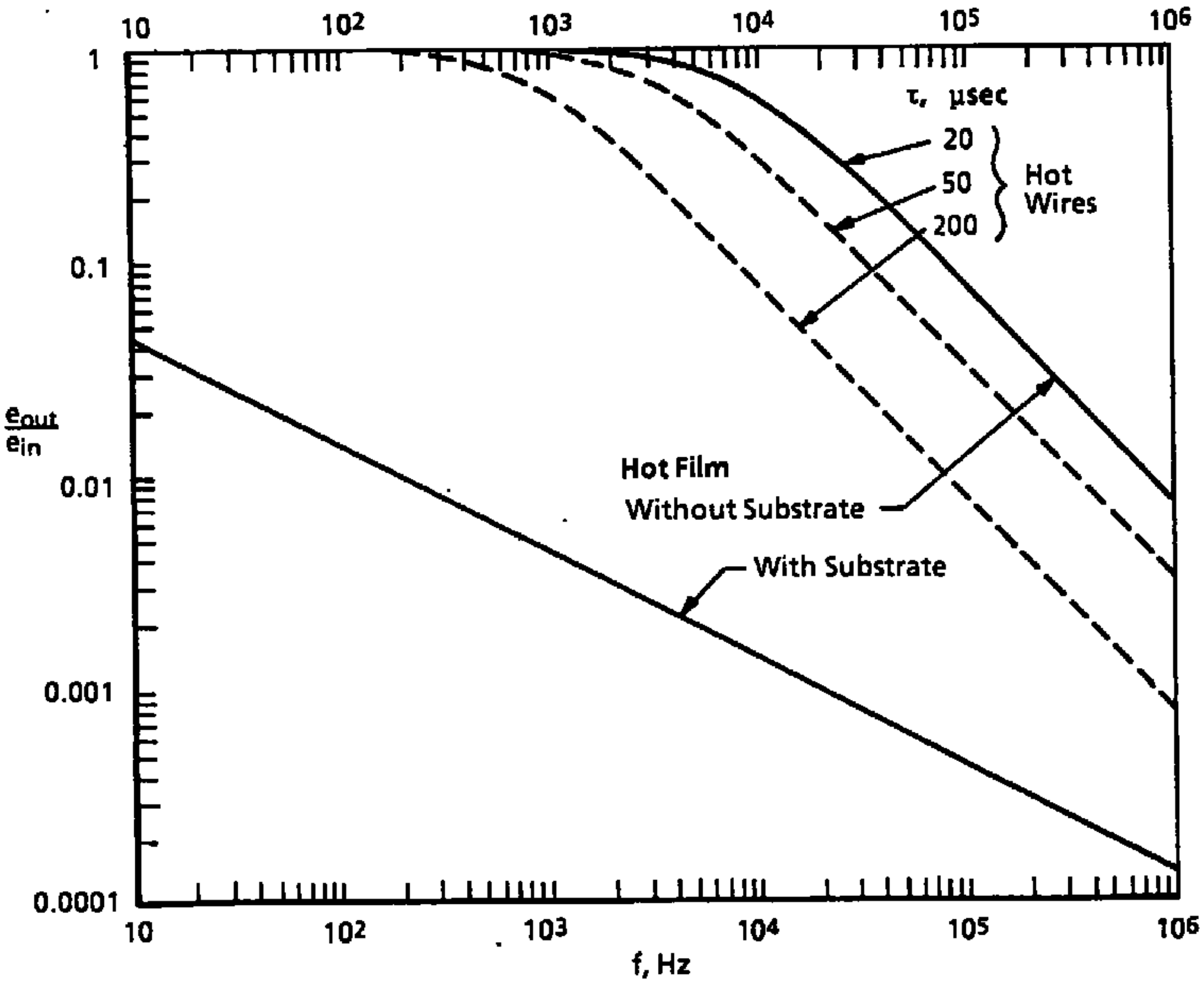

Figure 5. Example of a film probe response with and without substrate, and comparison with hot-wire probes.

LIST OF SYMBOLS

A_f: Film surface area

A_{hw}: Hot-wire surface area

A'_w: Morkovin's overheat parameter (Reference 13)

$\bar{A}$: Non-dimensional amplitude of film temperature fluctuation

B: Non-dimensional ohmic heating parameter (eq. (16))

c: heat capacity (energy per degree per unit mass)

d: Film thickness

d_w: Wire diameter

D: Probe diameter

d( ): differential (small) fluctuation

e: Mean film voltage

e': Film voltage fluctuation

e'_n: Net film voltage fluctuation

e_i: Film sensitivity coefficient

$\bar{E}$: non-dimensional input amplitude

f: dimensionless convection transfer parameter (eq. (15))

G: Factor expressing sensitivity to fluctuations (eq. (78))

$\bar{G}$: Modified form of G (eq. (80))

h: film height

i: electric current

I: non-dimensional current fluctuation amplitude i'/i

k: thermal conductivity

l: wire length

L: penetration length of film heating into substrate (eq. (28))

L_p: film probe length

L_c: A characteristic length

m: Exponent in temperature dependence of conductivity

M: Mach number

n: Exponent in the temperature dependence of viscosity

N: Nusselt number

$\bar{N}$: compound Nusselt no. (includes conduction losses)

N_M: logarithmic derivative of N relative to M (eq. (61))

N_p: Nusselt no. for convection on the probe body surface

N_r: logarithmic derivative of N relative to r (eq. (61))

N_{Re}: logarithmic derivative of N relative to Re (eq. (61))

Q: loss factor (eq. (51), (92))

r: overheat parameter = $(R-R_e)/R_e$

$\bar{r}$: modified overheat parameter (eq. (81))

r_p: radius of probe body

r_R: material electrical resistivity

R: film resistance

R_r: Reference film resistance (at i = 0, and 0°C)

Re: Reynolds number, based on stagnation viscosity

Re': unit Reynolds number; also, fluctuation in Re

Re_h: Reynolds number based on film height h

t: time

$\tilde{t}$: dimensionless time (eq. (9))

T: Temperature (of film, if unsubscribed)

T_r: film reference temperature (taken = 0)

u: flow velocity

w: film width

x: distance from film, in substrate

α: temperature coefficient of resistance

β: substrate material diffusivity (=$k/c\rho$)

δ: function of the flow Mach no. (eq. (64))

ϵ: film total heat capacity (energy per degree)

η: temperature recovery factor

η_M: logarithmic derivative of η relative to M (eq.(61))

η_{Re}: logarithmic derivative of η relative to Re (eq. (61))

θ : non-dimensional temperature (of film, if unsubscribed) see eq. (12)

μ: viscosity

ρ: density

τ: intrinsic time constant of film (eq. (41))

$\bar{\tau}$: non-dimensional τ (eq. (37), (39))

ϕ: phase angle

ω: angular frequency

$\bar{\omega}$: non-dimensional angular frequency = $\omega\beta/A_f$

$(\ )_e$: at recovery temperature conditions

$(\ )_f$: pertaining to the film

$(\ )_o$: at stagnation conditions

$(\ )_s$: pertaining to the substrate

$(\ )_w$: pertaining to a hot wire

$(\ )'$: fluctuating quantity (same as d( )) (note Re' exception).

$\hat{(\ )}$: non-dimensional fluctuation